THE INTERNAL BOUNDARY LAYER - A REVIEW

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Abstract. A review is given of relevant work on the internal boundary layer (IBL) associated with:

(i) Small-scale flow in neutral conditions across an abrupt change in surface roughness.

(ii) Small-scale flow in non-neutral conditions across an abrupt change in surface roughness, temperature or heat/moisture flux.

(iii) Mesoscale flow, with emphasis on flow across the coastline for both convective and stably stratified conditions.

The major theme in all cases is on the downstream, modified profile form (wind and temperature), and on the growth relations for IBL depth.

1. Introduction

Internal boundary layers (IBLs) in the atmosphere are associated with the horizontal advection of air across a discontinuity in some property of the surface. Studies usually specify the surface forcing in terms of a step change in surface roughness, temperature or humidity, or in the surface flux of heat or moisture. Several classes of IBL problems can be readily identified in the literature, concerning both laboratory and atmospheric flows, and related both to the thermal stability characteristics and horizontal scale of the flow.

Earlier work (from the late fifties to the mid-seventies) was concerned mainly with the problem of neutral flow across a step change in surface roughness. Almost without exception, this involved flows confined to the inner layer, i.e., to the wall region in laboratory flows and to the surface layer in the atmospheric case. Main interest involved the subsequent development downstream of the modified wind profile, the response of the turbulent field, and the growth of the IBL itself. Later in this period, attention turned to the effects of thermal stratification upon the flow across a roughness change, and to the growth and structure of the IBL related to step changes in surface heat flux and temperature. Throughout this period the main emphasis was on the small-scale aspects of the flow, i.e., on relatively small downwind fetches and where, in the atmosphere for example, the IBL was confined to the atmospheric surface layer of the advected planetary boundary layer. Such a constraint allowed for several simplifying assumptions in analytical and numerical treatments, and confined the downstream fetch to maximum values of about 1 km.

In more recent times (the mid-seventies to the present), the emphasis moved from the micrometeorological (and local advection) problem associated with relatively small fetches to mesoscale advection, and in particular to the effects of buoyancy and the development of the thermal IBL towards an equilibrium boundary layer far downstream of the leading edge (relatively large fetches). The main topic has been on the growth of the convective thermal internal boundary layer (TIBL) at a coast, mainly because of practical concern on the influence of the IBL on coastal pollution from industrial sites located in the coastal region. Less attention has been given to the parallel problem of a stably stratified IBL, whose full development may require fetches of several hundreds of kilometers.

In the case of small-scale flow and the neutral IBL responding to a change in surface roughness (specified in terms of the aerodynamic roughness length, z_0), analytical solutions were provided in studies by Elliott (1958), Taylor (1962), Panofsky and Townsend (1964), Townsend (1965), Plate and Hidy (1967), Taylor (1969a) and Mulhearn (1977); – see also Panchev *et al.* (1971) for a summary of Soviet studies. Numerical studies included those of Peterson (1969), Shir (1972), Rao *et al.* (1974a) and Beljaars *et al.* (1987). Observational studies are relatively few, but include – (i) wind tunnel experiments described by Taylor (1962), Antonia and Luxton (1971, 1972). Schofield (1975), Antonia and Wood (1975) and Mulhearn (1976; 1978); and (ii) atmospheric studies made by Bradley (1968), Panofsky and Petersen (1972), Petersen and Taylor (1973) and Munro and Oke (1975).

For the non-neutral, IBL response to a z_0 change, and to step changes in surface temperature and heat flux in particular, analytical solutions have been presented by, e.g., Sutton (1934, 1953), Philip (1959), Townsend (1965); numerical solutions by, e.g., Taylor (1970, 1971), Rao *et al.* (1974b), Rao (1975) and wind-tunnel observations by Antonia *et al.* (1977). Extensions of the small-scale approach to the mesoscale, and hence to the deeper IBL, have been discussed by Taylor (1969b) and Larsen *et al.* (1982).

In the case of mesoscale flow, and specifically the thermal IBL at the coast, models for the convective IBL and growth relations have been discussed by Venkatram (1977, 1986), Stunder and Sethuraman (1985), Bergstrom (1986), Hsu (1986), supported by observational studies of, e.g., Echols and Wagner (1972), Raynor *et al.* (1979), Gamo *et al.* (1982), Bergstrom *et al.* (1988) and Durand *et al.* (1989). For the stably-stratified thermal IBL, Mulhearn (1981), Hsu (1983, 1989) and Doran and Gryning (1987) compared observations of the IBL depth with simple formulations. Garratt (1987) compared the predictions of a numerical model with those of a physially based model for IBL depth, and detailed aircraft observations were described by Garratt and Ryan (1989).

In general, little attention has been given to the problem of the low-level thermal wind and its impact on IBL development, i.e., the effects associated with the horizontal temperature structure, and the implied modifications to the horizontal pressure field. Such a problem will be relevant only at the mesoscale and larger horizontal scales, and be associated with sea-breezes in the coastal example, and non-classical mesoscale circulations (or 'inland sea breezes') far from the coast in the presence of the relevant surface step changes. Mesoscale numerical models with appropriate boundary-layer parameterization schemes could shed light on this, and a few studies to date have indicated co-existence of both sea-breeze circulations and the IBL (e.g., Garratt, 1987; Physick *et al.*, 1989).

In the present paper, we review knowledge of the atmospheric IBL, with emphasis on the modification of vertical profiles of mean flow properties, and IBL growth relations, under a range of conditions.

2. Problems Relevant to IBL Growth

Solutions to the equations of motion representing boundary-layer growth over a suddenly accelerated plate (motion in the plane of the plate) and in parallel flow over a stationary flat plate, and to the diffusion equation for one- and two-dimensional flow, are relevant to the problem of IBL growth.

2a. Boundary-layer growth over a suddenly accelerated flat plate

Schlichting (1979) showed how the Navier–Stokes equations of motion could be simplified for the special case of a suddenly accelerated plate (motion in the plane of the plate) and unsteady, parallel laminar flow. If the plate attains instantaneously a steady velocity u_∞, then the resultant equation for the velocity response of the air above the plate can be written

$$\partial u/\partial t = \nu\, \partial^2 u/\partial z^2 , \tag{1}$$

where u is horizontal velocity, t is time, z is height above the surface and ν is the viscosity. This differential equation is identical with the equation of heat conduction which describes the propagation of heat in the space $z > 0$ when, at time $t = 0$, the wall $z = 0$ is suddenly heated to a temperature which exceeds that of the surroundings (see below). The solution to Equation (1), with $u = u_\infty$ at $z = 0$, $u = 0$ at $z = \infty$, is

$$u = u_\infty(1 - \mathrm{erf}(z/2(\nu t)^{1/2})) , \tag{2}$$

which has direct analogy to the case of flow over a flat plate on the one hand, and to the problem of heat diffusion to or from a horizontal plane on the other. Defining the top of a boundary layer (h) where $u = 0.01u_\infty$ (recalling that u_∞ is the motion of the plate), Equation (2) implies

$$h \propto (\nu t)^{1/2} . \tag{3a}$$

2b. Boundary-layer growth over a flat plate

For the laminar boundary layer over a thin flat plate, the solution to the Navier-Stokes equations is not so straightforward. Schlichting showed how solutions can be obtained using a nondimensional height scale suggested by the solution to Equation (1), viz., $z(u_\infty/\nu x)^{1/2}$ with $x = u_\infty t$, where u_∞ is the free-stream velocity. Several depth scales can be used to characterize the growth of the boundary layer; for example, based on the analogy to the solution given by Equation (2). Schlichting defined the boundary-layer thickness (h) as that height for which $u = 0.99u_\infty$. This gave

$$h \propto (\nu x/u_\infty)^{1/2}, \tag{3b}$$

so that the laminar boundary layer grows as $x^{1/2}$ for x not too small.

In contrast, consideration of solutions to the momentum equation for a turbulent boundary layer above a smooth flat plate showed (e.g., Schlichting, 1979, p. 638)

$$h \propto (\nu/u_\infty)^{0.2}x^{0.8}. \tag{4}$$

2c. Solutions of the diffusion equation

Sutton (1934 and 1953) showed how solutions of the heat diffusion equation for both one- and two-dimensional problems were relevant to the growth of an inversion in offshore flow. In the first instance, he investigated heat diffusion to the atmosphere over a homogeneous surface of fixed temperature (θ_0); this is directly analogous to Equation (1), and represented by

$$\partial\theta/\partial t = K\,\partial^2\theta/\partial z^2, \tag{5}$$

where K is a constant eddy diffusivity. For an atmosphere with an initial uniform temperature (θ_i) in the vertical, the distribution of temperature (θ) as a function of time is given by (see analogous solution to Equation (2)),

$$\theta - \theta_0 = (\theta_i - \theta_0)\,\mathrm{erf}(z/2(Kt)^{1/2}). \tag{6}$$

Sutton applied this to the case of offshore flow with a steady wind u constant with height, with continental air initially at temperature θ_i, and a sea-surface temperature of θ_0. The equation becomes

$$u\,\partial\theta/\partial x = K\,\partial^2\theta/\partial x^2, \tag{7}$$

with solution

$$\theta - \theta_0 = (\theta_i - \theta_0)\,\mathrm{erf}(z(u/4Kx)^{1/2}). \tag{8}$$

In practice, Sutton took the influence of turbulent mixing on the θ profile to extend to a finite height defined arbitrarily as the height where the error function equals 0.1 (cf. Schlichting's criterion for the velocity profile given above); this gave the depth of the inversion (or thermal internal boundary layer) h as

$$h = 0.18(Kx/u)^{1/2}. \tag{9}$$

The square-root dependence of h on x is important to note. A similar dependence was found in the case of laminar boundary-layer growth over a flat plate (see e.g. 3b); it occurs also in the diffusion of a pollutant cloud, with h replaced by cloud width (the concentration diffusion equation is analogous to Equation (7) with solutions obtained for a range of source specifications).

In Philip's (1959) paper on the theory of local advection, solutions of the two-

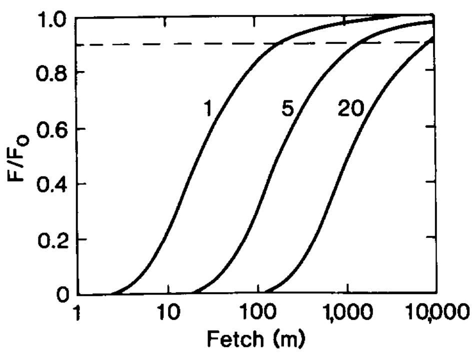

Fig. 1. The rate of adjustment of vertical fluxes (F/F_0) as a function of fetch and height (shown in metres against each curve) – from Dyer (1963). Here, F_0 is the downstream surface flux at large fetch.

dimensional atmospheric diffusion equation (cf. Equation (7)) were discussed for a range of surface boundary conditions. In this case, he started with

$$u\,\partial C/\partial x = -\partial F/\partial z\,, \tag{10a}$$

with C representing a concentration, and the vertical flux written as,

$$F = -K\,\partial C/\partial z\,. \tag{10b}$$

Solutions were obtained by setting $u = u_1(z/z_1)^m$ and $K = K_1(z/z_1)^n$ (subscript 1 refers to reference values) with the appropriate boundary conditions. Flow modifications for step changes in C_0 (surface concentration), F_0 (surface flux) and a linear combination of these (termed a radiation boundary condition) were derived and discussed.

The practical application of this theory to the micrometeorological problem of adjustment of profiles and eddy fluxes to surface changes, and the fetch-height ratio, was discussed by Dyer (1963). He considered the problem of completely dry air moving from a non-evaporating regime over an area where the evaporation is everywhere constant – thus the surface change is one where the surface flux increases from zero to a value of F_0 representative of large values of fetch. The results (based on Philip's solutions) were presented in terms of the ratio F/F_0 and its dependence on fetch and height downwind of the leading edge (or surface discontinuity). Figure 1 shows these, and indicates the values of fetch and height for the 90% level of adjustment sometimes used in micrometeorological considerations (this corresponds to an inner equilibrium layer, found within the IBL itself). Extensions of the above results were presented by Dyer and Garratt (1978) for the case of a meteorological tower situated within the IBL for onshore flow near the coast.

3. Definition of the IBL

Figure 2 shows in schematic form the concept of an internal boundary layer, and the inner equilibrium layer, for a step change in roughness, surface temperature,

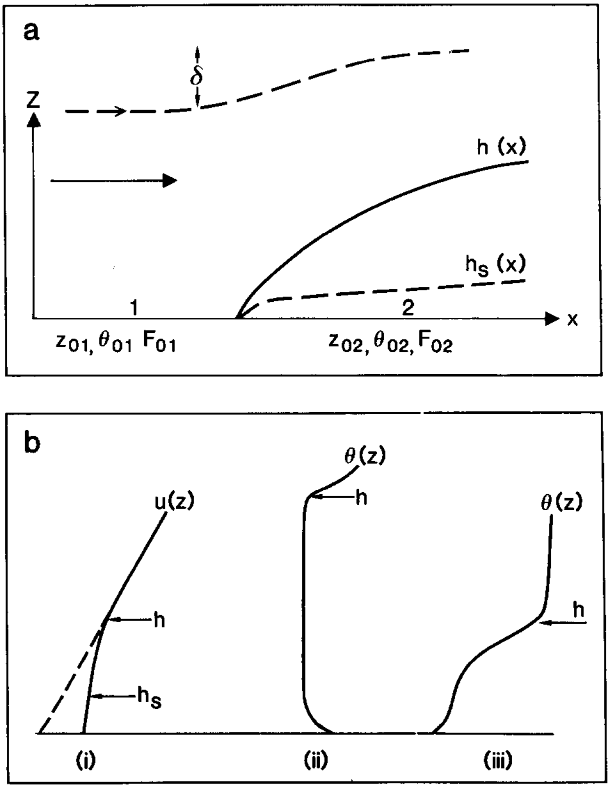

Fig. 2. (a) Schematic representation of the IBL $h(x)$ and inner equilibrium layer $h_s(x)$ downstream of a step change in roughness (z_0), temperature (θ_0) and heat or moisture flux (F_0). Streamline displacement, δ, is also shown. (b) Vertical profiles at a distance x downstream of the discontinuity – (i) wind profile for neutral flow across a z_0 change; (ii) θ profile for an unstable IBL and θ_0 change; (iii) θ profile for a stable IBL and θ_0 change.

or surface flux based in part on observations at both the microscale and mesoscale. Typical profiles of wind speed and potential temperature are also shown to illustrate the presence of an IBL. This is not always readily identified or defined, particularly in the case of neutral flow. In boundary-layer flow over a plate, the depth was defined by Schlichting where $u = 0.99u_\infty$; he also defined a displacement thickness and a momentum thickness, the latter being a height scale related to the total loss of momentum due to the boundary layer compared to potential flow. Both these scales were simple fractions of the boundary-layer depth.

In the case of a response to z_0 changes, the IBL has been defined in a variety of ways – e.g., by $\partial u/\partial z$ discontinuities or wind-profile 'kinks' (Elliott, 1958; Panofsky and Townsend, 1964; Bradley, 1968; Antonia and Luxton, 1971, 1972). In fact, Shir (1972) defined a velocity IBL (h_1) whose top occurred at $u = 0.99u_\infty$ and a stress IBL (h_2) where τ (at h_2) $= 0.99\tau_0$ (τ_0 being the upstream surface stress); typically, $h_1 < h_2$ since velocity profiles are found to adjust more slowly than stress. Rao *et al.* (1974a) defined an IBL (depth h) where $u = 0.99u_\infty$ and an equilibrium layer (depth h_s) at the top of which a 90% level of adjustment of the local stress to the underlying (downstream) surface had occurred (see earlier discussion of Dyer, 1963); typically, $h_s \ll h$.

In contrast to the above, the definition of the TIBL is generally less ambiguous. In the convective case in the coastal region, its top is readily identified where $\partial\theta/\partial z$ has a discontinuity (Raynor *et al.*, 1975) or at the top of a well-mixed layer (Venkatram, 1977). However, Gamo *et al.* (1982) and Durand *et al.* (1989), based on airborne measurements, found a minimum in turbulent kinetic energy above the gradient 'discontinuity' or θ inversion, although in the numerical studies of both Arritt (1987) and Durand *et al.* (1989), the minimum was found to more or less coincide with the IBL top inferred from the θ profile.

In the stable case, temperature and humidity profiles show similar gradient discontinuities near the IBL top; generally these occur at the base of a layer with small gradients only (Garratt, 1987; Garratt and Ryan, 1989). In analogy with the study of Arritt, Garratt (1987) found in his numerical simulations that the IBL top coincided with a minimum in the eddy diffusivity.

To summarise, for neutral small-scale flow over a z_0 change, the IBL is the layer within which significant changes from upstream conditions occur, in $u(z)$ and $\tau(z)$ – discontinuities in $\partial u/\partial z$ in particular allow its top to be identified. Within the IBL there exists an equilibrium layer often defined in terms of a 90% level of adjustment in the stress (or other vertical flux in the non-neutral case). Far downstream of the leading edge, in neutral conditions, the inner equilibrium layer is characterized by a logarithmic profile form, traditionally referred to as the 'constant-flux layer' though in reality the momentum flux or friction velocity will decrease with height (the existence of a logarithmic wind profile does not depend on the assumption of constant flux). For non-neutral flow situations the above criteria could be applied to defining h, with additional information available from temperature and heat-flux profiles. In the case of the TIBL at larger scales, the IBL top is readily identified with an elevated inversion in the convective case, and with the top of a surface-based inversion in the stable case.

4. Small-scale Flow – Response to Roughness Changes

4a. Neutral flow – observations

Observations of the development of an IBL downstream of a roughness change reveal the following features:

(i) Above the IBL (defined as described in the previous section), the flow field is characteristic of the upstream conditions, except for a displacement δ of the outer flow field (the streamlines) required by continuity.

(ii) Very near the ground, an inner or 'equilibrium' layer exists where the wind profile has completely adjusted to the local boundary conditions.

(iii) Above this equilibrium layer, and within the IBL as a whole, there exists a blending layer (Plate, 1971) in which the velocity distribution gradually changes from the logarithmic form of the downstream roughness to that of the upstream one.
(iv) At large distances from either side of the discontinuity, the shear stress at the surface is adjusted to that of flow above a uniform surface.

Observations from wind-tunnel, pipe and duct experiments (e.g., Jacobs, 1939; see also Schlichting, 1979, p. 658; Logan and Jones, 1963; references in Plate, 1971; Antonia and Luxton, 1971 and 1972; Schofield, 1975; Antonia and Wood, 1975; Mulhearn, 1976, 1978) have concentrated on the development of the IBL and its internal mean and turbulent structure. Studies have involved both zero and adverse pressure gradient conditions. Atmospheric observations, mainly emphasising the modification to the low-level wind profiles, have been reported by Stearns (1964) and Stearns and Lettau (1963) based on the bushel-basket experiments over the ice of Lake Mendota in the USA, by Panofsky and Petersen (1972) and Petersen and Taylor (1973) based on the Risø tower observations, by

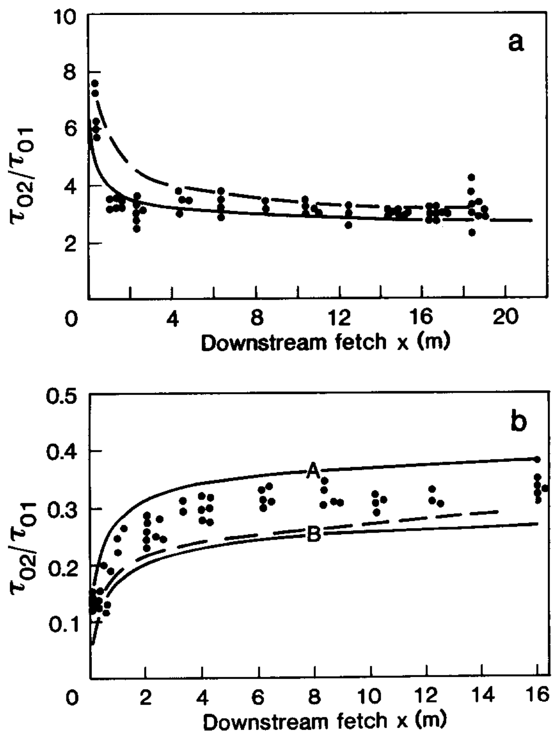

Fig. 3. (a) Variation of surface stress with fetch for a smooth-to-rough transition – data from Bradley (1968); dashed curve from Panofsky and Townsend (1964) and continuous curve from Rao *et al.* (1974a). (b) As in Figure 3a for a rough-to-smooth transition; curves A and B, from Rao *et al.* (1974a), have z_{02} equal to 2×10^{-5} m and 2×10^{-6} m, respectively.

Munro and Oke (1975) for flow downwind of the leading edge of a wheat crop, and by Bradley (1968). Bradley's data, which include simultaneous measurements of velocity profiles and surface stresses at several positions relative to a discontinuity separating grass and tarmac, and tarmac and artificial roughness, are ideal for comparison with theoretical and numerical results.

The results for surface stress, both for flow from smooth to rough, and for flow from rough to smooth surfaces, are shown in Figure 3, together with curves to be discussed later in the section. Both sets of data show a trend towards equilibrium stress values (as do the curves), but two features are of particular interest – in the smooth-to-rough case, the stress initially increases to about twice the final (large fetch) value. In the rough-to-smooth case, the initial stress decreases to about one half of the final value; theoretical and numerical results generally capture this.

Results for velocity profiles are shown in Figure 4, together with numerical results of Rao *et al.* (1974a) to be discussed later. Several features are of interest:

(i) There is a systematic shift of the profiles, left or right, as fetch increases, and a systematic increase in the upper point of the modified profile as fetch increases – the latter representing the increase in the IBL depth with fetch.

(ii) The mean velocity profile is virtually unchanged above this upper point. Plate (1971) interpreted this as evidence for only a minor deflection of the streamlines, implying that δ is small (this does depend however on the relative change in the magnitude of the roughness).

(iii) There is only a small portion of the profile, at any value of x, in which the velocity distribution deviates from the low-level logarithmic form characteristic of downstream conditions, or the upper logarithmic form above the IBL top.

The purpose of analytical and numerical treatments of the problem is to reproduce the above observations, and to quantify the growth equation for the IBL.

4b. Neutral flow – analytical theory

For small-scale problems, and neglecting details too close to the discontinuity (e.g., Peterson, 1972), pressure is taken to be constant everywhere (Rao *et al.*, 1974a actually incorporated pressure vartiations in their numerical study). The governing equations, with w the vertical velocity, simplify to (see Plate, 1971, e.g.)

$$u\, \partial u/\partial x + w\, \partial u/\partial z = \rho^{-1}\, \partial \tau/\partial z\,, \tag{11}$$

and the continuity equation,

$$\partial u/\partial x + \partial w/\partial z = 0\,. \tag{12}$$

To solve for $u(x, z)$ requires an equation for τ and suitable boundary conditions; in general, numerical methods or approximate techniques are used. The latter

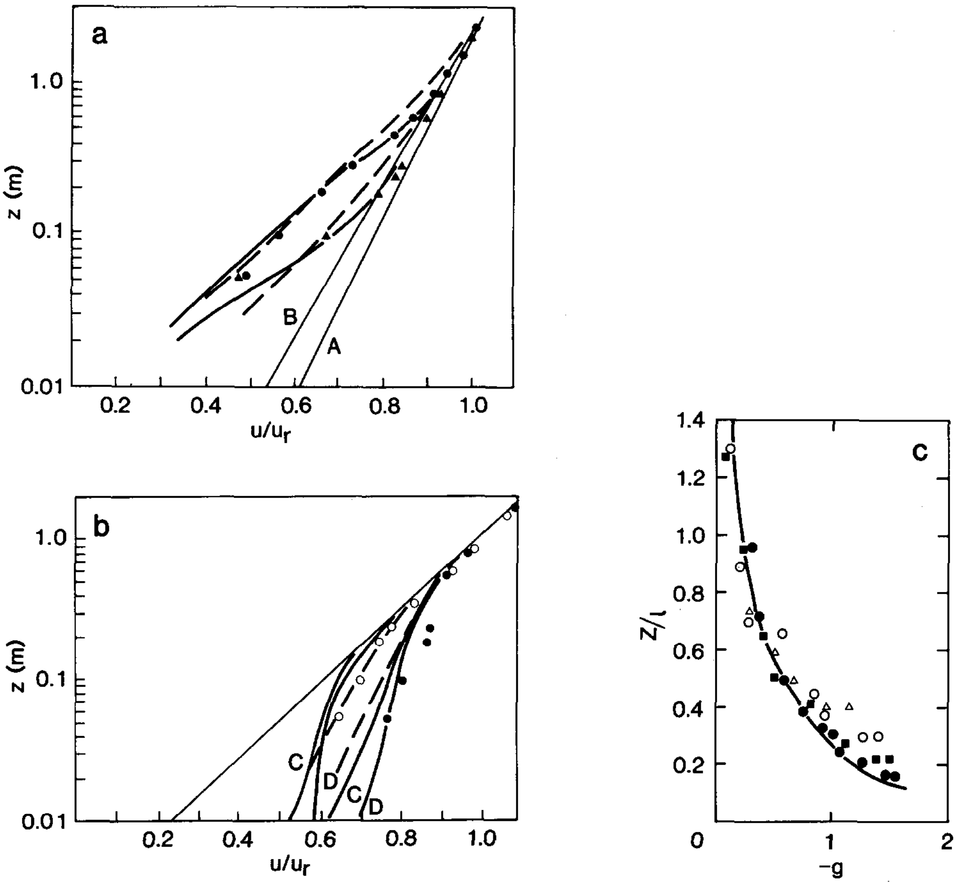

Fig. 4. (a) Adjustment of wind profile downwind of a smooth-to-rough transition ($z_{01} = 2 \times 10^{-5}$ m; $z_{02} = 0.0025$ m) – data from Bradley (1968); closed triangles have $x = 2.32$ m, closed circles have $x = 16.42$ m. Dashed curves from Panofsky and Townsend (1964) and continuous curves from Rao *et al.* (1974a). Curves A and B are the upstream profiles assumed by Panofsky and Townsend, and Bradley, and by Rao *et al.*, respectively. In the abscissa, u_r is a reference velocity measured at $z = 2.2$ m. (b) As in Figure 4a for a rough-to-smooth transition (z_0 values reversed); open circles have $x = 2.1$ m and closed circles have $x = 12.2$ m. Curves C and D (continuous) have $z_{02} = 2 \times 10^{-5}$ m and 2×10^{-6} m, respectively. In the abscissa, u_r is a reference velocity measured at 1.125 m. (c) Comparison of normalised wind profiles between the theoretical predictions of Mulhearn (1977) and wind-profile data of Bradley (1968) for a smooth-to-rough transition – see Equation (17a). The plotted data have a range of fetches indicated by different symbols. Here $g(\eta) = ku'/u_0$, with $\eta = z/l$; evaluation of l, u_0 and u' is described in the text. From Mulhearn (1977).

uses analytical theory where the crucial requirement is to represent the relation between stress and the velocity profile so as to solve for $\tau_0(x)$, $u(x, z)$ and $h(x)$. The concept of the self-preservation of velocity and stress changes (also temperature and concentration when considering different boundary conditions) is important here, used both in the Karman–Polhausen method of integral constraint on the momentum balance of the whole IBL (Schlichting, 1979), and in direct application of the equations of motion.

The integral method is based on the integration of Equation (11) from the surface to h, substituting for w from Equation (12), to give

$$\frac{\partial}{\partial x}\left\{\int_0^h u^2\,\mathrm{d}z\right\} - u_h\frac{\partial}{\partial x}\left\{\int_0^h u\,\mathrm{d}z\right\} = \tau_h - u_{*2}^2\,, \tag{13}$$

where subscripts 1 and 2 refer to upstream and downstream surfaces, respectively. This approach was used by, e.g., Elliott (1958), Panofsky and Townsend (1964) and Plate and Hidy (1967). In the above, u_h is given by u_1 at $z = h - \delta$, with τ_h equated to the appropriate upstream stress. The velocity distribution $u_2(z)$ is required, and is generally assumed to be given by

$$u_2(z)/u_{*2} = k^{-1}\ln(z/z_{02}) + f(z/h)\,, \tag{14}$$

where $f(z/h) = 0$ for $z/h \ll 1$. Various approaches can be summarised as follows,

(i) Elliott (1958) set $f(z/h) = 0$ and $\delta = 0$, basically implying constant stress throughout the IBL (a logarithmic profile throughout) and a stress discontinuity at $z = h$, the stress changing from u_{*2}^2 to u_{*1}^2 across h. The main result was to quantify a relation for the IBL growth (see later discussion).

(ii) Plate and Hidy (1967) modified the above by taking a non-zero δ value.

Both (i) and (ii) probably give satisfactory results if the blending region is thin, i.e., for small x only. At large x, this will not be the case; over a significant part of the IBL, above an inner or equilibrium layer, the function $f(z/h)$ will be significant.

(iii) Panofsky and Townsend (1964) took a form of $f(z/h)$ given by

$$f(z/h) = (u_{*1} - u_{*2})z/uh\,, \tag{15}$$

which is consistent algebaically with the self-preserving relation $u_* = kz\,\partial u/\partial z$, and a linear variation of u_* across the IBL.

(iv) Townsend (1965, 1966) and Mulhearn (1977) used Equations (11) and (12) directly, not in integrated form, and introduced self-preservation concepts or similarity arguments to improve the specification of the blending function, i.e., with dependence upon a length scale l only. Mulhearn actually derived analytical forms for the nondimensional changes in velocity (and temperature and concentration), whilst these were assumed by Townsend. Self-preservation in velocity and stress changes was assumed; consequently, the downstream velocity can be expressed as

$$u_2(z) = u_1(z) + \Delta u(\delta) + u'\,, \tag{16}$$

where Δu is related to streamline displacement (δ) and u' to flow acceleration within the IBL. Both of the above authors expressed Δu as $u_*\delta/kz$ and introduced the similarity hypotheses, for u,

$$u' = (u_0/k)g(\eta)\,, \tag{17a}$$

and for τ,

$$\tau = u_{*1}^2 + (u_{*2}^2 - u_{*1}^2)G(\eta)\,, \tag{17b}$$

where $\eta = z/l$, l and u_0 being length and velocity scales depending on x only. The stress-gradient relation $u_* = kz\,\partial u/\partial z$ allows g and G to be related and to be expressed as functions of η, and the scales l and u_0 are found from the facts that g and G are functions of η only, and that the velocity profile assumes the logarithmic form close to the surface (see also Blom and Wartena, 1969).

Comparison of the theoretical results of Panofsky and Townsend (1964) with Bradley's observations of stress variation is shown in Figure 3; this reveals, in the case of smooth-to-rough flow, that at small x/z_0, the data exhibit a more rapid variation of surface stress than any theory. In the case of rough-to-smooth flow, the stress variation is well described although absolute values disagree; this is due, in part at least, to the sensitivity to the chosen z_0 value (Bradley, 1968; Nemoto, 1972). The comparison of velocity profiles can be seen in Figure 4a, b; their form and relative displacement have considerable similarity, although the observed velocity gradient discontinuity appears sharper. Overall however, the height of the modified region appears to be lower than the theory predicts. Figure 4c shows comparisons with Bradley's data of the predictions of Mulhearn (1977) for non-dimensional wind profiles, based on Equations (16) and (17). In Equation (17a), u_0 was taken as $1.4u_{*1}$, l was given by $\ln(l/z_{01}) = 12.4$ and u' evaluated from Equation (16) based on the observed profiles. The comparisons tend to support the self-preservation assumptions on the downstream profiles, and show good agreement except for small η.

The crucial shortcoming of the theories discussed above, including the diffusion approach of, e.g., Philip (1959), and studies related to temperature changes at the surface (see later section), e.g., Taylor (1970, 1971), is the use of a mixing-length and/or eddy-diffusivity assumption. In a sense, this is a failure to represent properly the relation between stress and velocity gradient in a transition or non-equilibrium situation, where relations such as $u_* = kz\,\partial u/\partial z$ are inappropriate (as evidenced in numerical studies to be discussed shortly). This can be overcome to a great extent by carrying an equation for τ, or turbulent kinetic energy, and solving the equations numerically.

4c. Neutral flow – numerical

An equation for τ is incorporated by utilising the turbulent kinetic energy (TKE) equation and a relation between stress and the TKE (e.g., Blackadar *et al.*, 1967;

Peterson, 1969; Panchev *et al.*, 1971; Shir, 1972; Petersen and Taylor, 1973; Huang and Nickerson, 1976), or using the full second-order turbulence equations (e.g., Rao *et al.*, 1974a) with suitable parameterization of the third-order terms. In the former case, the TKE equation, with E being the TKE,

$$dE/dt = (\tau/\rho)\,\partial u/\partial z - \partial/\partial z(\overline{wE}) - \epsilon\,, \tag{18}$$

is used with the assumptions that $\tau/\rho = \alpha E$ and $\epsilon = (\tau/\rho)^{3/2}/\lambda$, λ being a length scale having a value of kz close to the surface. The vertical divergence term is parameterized by assuming a flux-gradient relation, with diffusivity identical to that assumed in the stress-velocity gradient relation.

The problem with this TKE approach is the use of stress and dissipation relations probably valid in constant-stress, equilibrium layers rather than in transitional flow regimes. The approach of Rao *et al.* (1974a, b) is to carry the time-dependent turbulence equations, for variance and covariance quantities (including that for τ), and approximate the third-order terms. For illustration, the results of Rao *et al.* (1974a) are given in Figure 3 for the surface stress variation with fetch for comparison with Bradley's data. Generally the stress distribution is better predicted than by any of the analytical methods. The numerical simulations also provided vertical profiles of stress and other turbulent statistics, typical of those available only in detailed wind-tunnel experiments (e.g., Mulhearn, 1978). A comparison of velocity profiles is shown in Figure 4a, b; quite good agreement is found although the numerical results differ from the mixing theories by showing a transitional velocity profile in the blending region with double curvature, similar to those actually observed. This result is best illustrated by introducing the non-

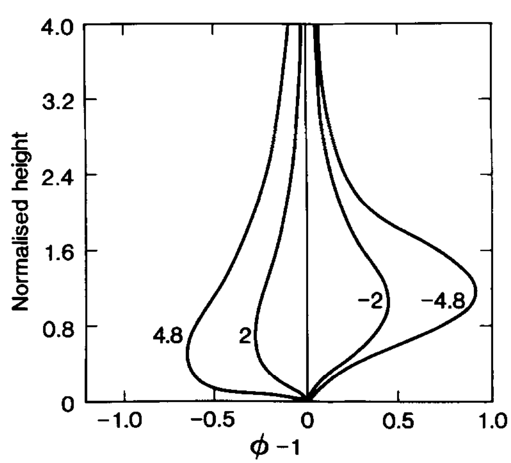

Fig. 5. Variation of the non-dimensional wind shear Φ with normalised height for several values of the roughness-change parameter $M = \ln(z_{01}/z_{02})$ at $x = 2$ m. Results are from the numerical simulations of Rao *et al.* (1974a).

dimensional gradient Φ given by

$$\Phi = (kz/u_*)\, \partial u/\partial z \tag{19}$$

whose variation with dimensionless height is shown in Figure 5. Values differing from unity, and hence from equilibrium values, reveal the effect of transitional flow upon Φ (and other similarity variables) and the non-validity of mixing-length assumptions based on $l = kz$ (which implies $\Phi = 1$).

4d. Roughness change – growth of IBL

The growth relation depends to some extent on how the modified region or IBL is defined (see Section 3). Observations of turbulent flow from smooth-to-rough, and rough-to-smooth, surfaces in the atmosphere (e.g., Bradley, 1968) generally are consistent with the turbulent boundary-layer growth over a smooth plate (e.g., Schlichting, 1979), viz., $h \propto x^{0.8}$. Wind tunnel data of Antonia and Luxton (1971 and 1972) on the other hand (for zero pressure gradient conditions) showed a $x^{0.79}$ dependence for smooth-to-rough flow and an $x^{0.43}$ dependence for rough-to-smooth flow. The slower growth in the latter case seemed to be related to the higher turbulence levels above the IBL, although this slower growth does not seem to be reflected significantly in atmospheric observations (also, see Jackson, 1976) or in semi-empirical formulations (except see Equation (20) below). Mention should also be made of the wind-tunnel study of Schofield (1975) for IBL growth in adverse pressure-gradient conditions. His data ($x/z_0 \approx 10^4$–10^5) for the smooth-to-rough transition, when combined with those of Antonia and Luxton ($x/z_0 \approx 10$–10^3) for both smooth-to-rough and rough-to-smooth transitions, gave $h \propto x$ approximately over the broader fetch range.

The data of Antonia and Luxton (1971) are compared with predictions of analytical theories in Figure 6a, and Bradley's (1968) data compared with numerical simulations in Figure 6b. Displacement of the curves usually results from different definitions of the IBL. Some studies have attempted to include explicitly the effect of roughness, or roughness change, into the formulation. For example, both Elliott (1958) and Wood (1982) suggested $h \propto x^{0.8} z_0^{0.2}$, with z_0 the greater of z_{01} and z_{02}, whilst Shir (1972) discussed a more general form for neutral flow

$$h = f_1(z_{01}/z_{02})\, x^{0.8 + f_2(z_{01}/z_{02})}\,. \tag{20}$$

According to Shir's numerical results, the function f_2 is slightly negative in rough-to-smooth flow, and close to zero for smooth-to-rough flow.

Some authors have utilised a diffusion analogue (or principle of limited diffusion rate) to evaluate the slope of the IBL, and hence a relation between h and x (e.g., Panofsky, 1973; Jensen, 1978; Larsen *et al.*, 1982). This uses the concept of zones of influence, with analogy between the zone influenced by the downstream roughness and the spread of a smoke plume from a ground-level source in uniform roughness (Miyake, 1965; Jackson, 1976; Panofsky and Dutton, 1984). In this, u_*

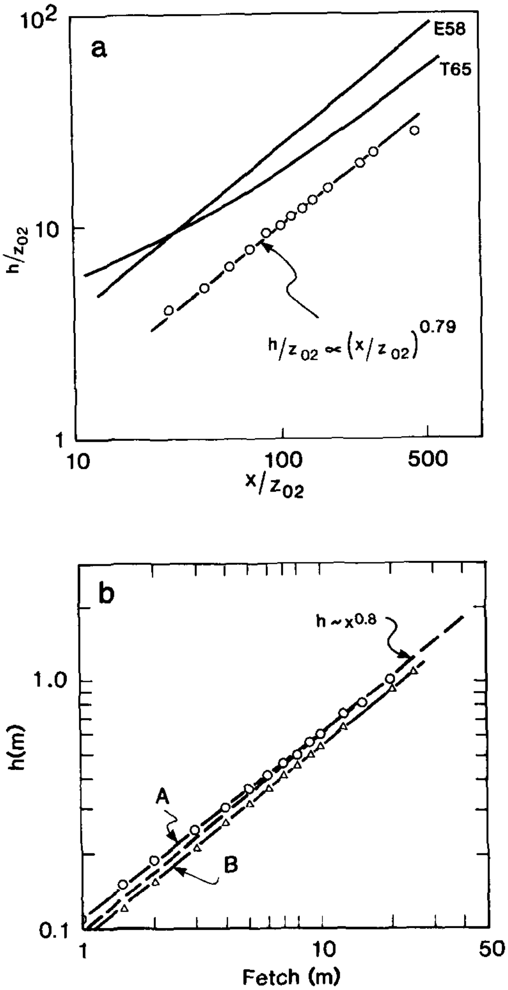

Fig. 6. (a) Growth of the IBL after a roughness change (smooth-to-rough) based on the wind-tunnel data of Antonia and Luxton (1971), and comparisons with the theories of Townsend (1965) [T65] and Elliott (1958) [E58]. From Antonia and Luxton (1971). (b) Growth of the IBL based on the model of Rao *et al.* (1974a) for comparison with the data of Bradley (1968) – the dashed curve; continuous curves (joining computed values) are for different values of M (curves A and B have $\ln(z_{01}/z_{02}) = 4.8$ and -4.8, respectively.

or the vertical velocity variance (σ_w^2) is assumed to determine the growth rate, with

$$dh/dx \propto \sigma_w/u(h) \propto u_*/u(h) \propto k/\ln(h/z_0)\,, \tag{21}$$

and, after integration,

$$(h/z_{02})(\ln(h/z_{02}) - 1) + 1 = A\,x/z_{02}\,, \tag{22}$$

with $A \approx 1$.

Analogous relations to Equation (22) were derived by Pasquill (1972) and Jackson (1976). Pasquill's approach was to consider the mean vertical displacement ($\bar{z}$) of passive particles diffusing in space at a given time after release from the ground. Using $d\bar{z}/dt = ku_*$, together with Lagrangian similarity arguments (after Batchelor), led to Equation (22), with h replaced by $\bar{z}$ and $A = k^2$. Good correspondence with Peterson's (1969) IBL growth relation was found with $h = 3\bar{z}$. In Jackson (1976), an extension of Miyake's (1965) theory led to a small correction term (for origin effects) on the left-hand side of Equation (22), with z_{02} replaced by $z_0' = 0.5(z_{01}^2 + z_{02}^2)^{0.5}$. The modified relation agreed satisfactorily with both atmospheric (Blackadar *et al.*, 1967; Bradley, 1968) and wind-tunnel measurements (Plate and Hidy, 1965; Antonia and Luxton, 1971, 1972) – see Jackson (1976) for additional references.

Recently, Walmsley (1989) considered several IBL depth formulae – due to Elliott (1958), Jackson (1976) and Panofsky and Dutton (1984), and compared their predictions with atmospheric data. The data taken from Jackson (1976), Peterson *et al.* (1979) and an unpublished source were generally confined to fetches less than about 200 m. Equation (22) was found to give the best predictions, with $A = 1.25k$ after Panofsky and Dutton (1984).

Finally, the similarity analyses of Townsend (1965, 1966) and Blom and Wartena (1969) lead to Equation (22), with h replaced by a general length scale l (see Section 4b) and $A = 2k^2$. In general, integral methods based on the neutral surface-layer approximation imply a relation between h and x of quite similar form to that expressed in Equation (22).

4e. Effects of Thermal Stratification

The effects of thermal stratification on IBL structure and growth in relation to a roughness change have often been studied in association with the impact of surface heat flux and temperature changes. This problem is discussed in Section 5. In the context of the IBL depth, a few studies do exist which help clarify the effects of thermal stratification on the IBL growth downstream of a roughness change. Echols and Wagner (1972) described observations of a shallow IBL formed inland of a coastal region, with evidence for a deeper layer by day compared to the night. Rao (1975) extended the numerical work of Rao *et al.* (1974a) to include thermal effects; thus with $h \propto x^n$, n was found to increase from 0.8 in neutral to about 1.4 in strongly unstable conditions, viz., a much more rapid growth when surface heating was present.

5. Small-scale Flow – Response to Changes in Surface Heat Flux and Temperature

5a. Observations

The micrometeorological data of Rider *et al.* (1963) and of Dyer and Crawford (1965) are probably best known in the context of the 'leading edge' or local advection problem, involving as they do the small-scale variation in flow properties downwind of a surface change in wetness (dry to wet). The first authors in fact compared their data with the diffusion-based theory of Philip (1959), but we shall use some of the data to illustrate the predictions of the temperature profile evolution in particular, based on analytical theory and numerical simulations. Other relevant atmospheric data have been described by Taylor (1970) for flow across the shoreline of one of the Great Lakes in the USA, and by Vugts and Businger (1977) for flow across a beach – both are related to air modification in the presence of a step change in surface temperature. In addition to the above, Antonia *et al.* (1977) described wind-tunnel data and the response of a turbulent boundary layer to a step change in heat flux at the surface. All the data allow some inferences to be drawn regarding the growth of the thermal IBL, with Antonia *et al.* (1977) finding support for a 4/5 power law found for the roughness change in neutral flow.

5b. Theory

Equations used in the roughness change analysis for neutral flow are augmented by a θ equation (compare with Equation (7)),

$$u\,\partial\theta/\partial x + w\,\partial\theta/\partial z = -(\rho c_p)^{-1}\,\partial H/\partial z\,, \tag{23}$$

with H, the vertical turbulent heat flux, represented by a flux-gradient relation.

The self-preservation or similarity approach of Townsend (1965) and Mulhearn (1977) discussed in the previous section was extended by these authors to study the response downstream of surface changes in heat flux and temperature. For example, the downstream θ profile ($\theta_2(z)$) can be expressed as

$$\theta_2(z) = \theta_1(z) + \theta'\,, \tag{24a}$$

where θ' is defined through the similarity hypothesis

$$\theta' = (\theta_c/k)g(\zeta)\,, \tag{24b}$$

with the nondimensional height $\zeta = z/l_c$. Likewise, the heat flux is written as

$$H = H_2 + (H_2 - H_1)G(\zeta)\,, \tag{25}$$

where H_1 and H_2 are the surface heat fluxes downstream and upstream of the discontinuity, respectively. As before, g and G can be related as functions of ζ, and scales l_c (length) and θ_c (temperature) are evaluated in analogy with the scales for velocity. Theoretical results were compared with observations of Rider *et al.* (1963), Dyer and Crawford (1965) and Blom (1970; for this reference see Mulhearn, 1977). Examples are given in Figure 7, both for the standard temperature profile (Figure 5 of Townsend, 1965) and for the normalised temperature profile demonstrating their self-preservation form (Figure 5 of Mulhearn, 1977; where ζ for the θ profile is analogous to η for wind). In the latter, the temperature

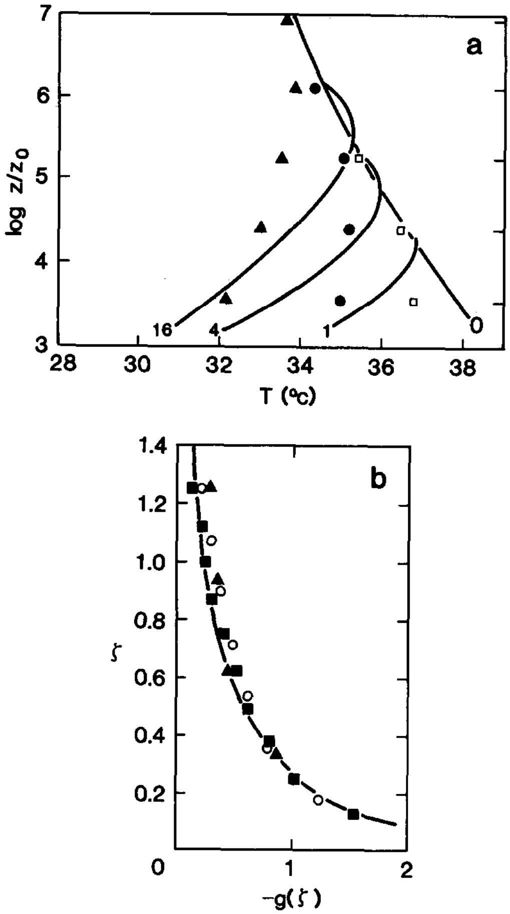

Fig. 7. (a) Comparison of predicted (Townsend, 1965) and observed temperature profiles (data of Rider *et al.*, 1963) downstream of a change in surface evaporation – x values are shown against each curve and experimental profile. The data are for $x = 1$ m (□), $x = 4$ m (●) and $x = 16$ m (▲). From Townsend (1965). (b) Comparison between the theoretical predictions of Mulhearn (1977) and the temperature data of Dyer and Crawford (1965), with symbols representing different values of fetch. Here $\zeta = z/l_c$ and $g(\zeta) = k\theta'/\theta_c$; evaluation of l_c, θ_c and θ' is described in the text. From Mulhearn (1977).

scale θ_c was taken as H_1/ku_{*1}, and l_c taken from $\ln(l_c/z_{01}) = 9.4$. The perturbation was calculated from the observed θ profiles in analogy to the velocity u'.

5c. Numerical

Panchev *et al.* (1971) described numerical studies by Nadejdina (1966, 1969) who used the TKE equation with a range of assumptions (similar to the approach of Peterson, 1969) to solve for $\theta(z)$ and $h(x)$. Taylor (1970, 1971), using a mixing length approach, described numerical simulations of flow modifications due to step changes in temperature and heat flux at the surface. In Taylor (1970), neutral and

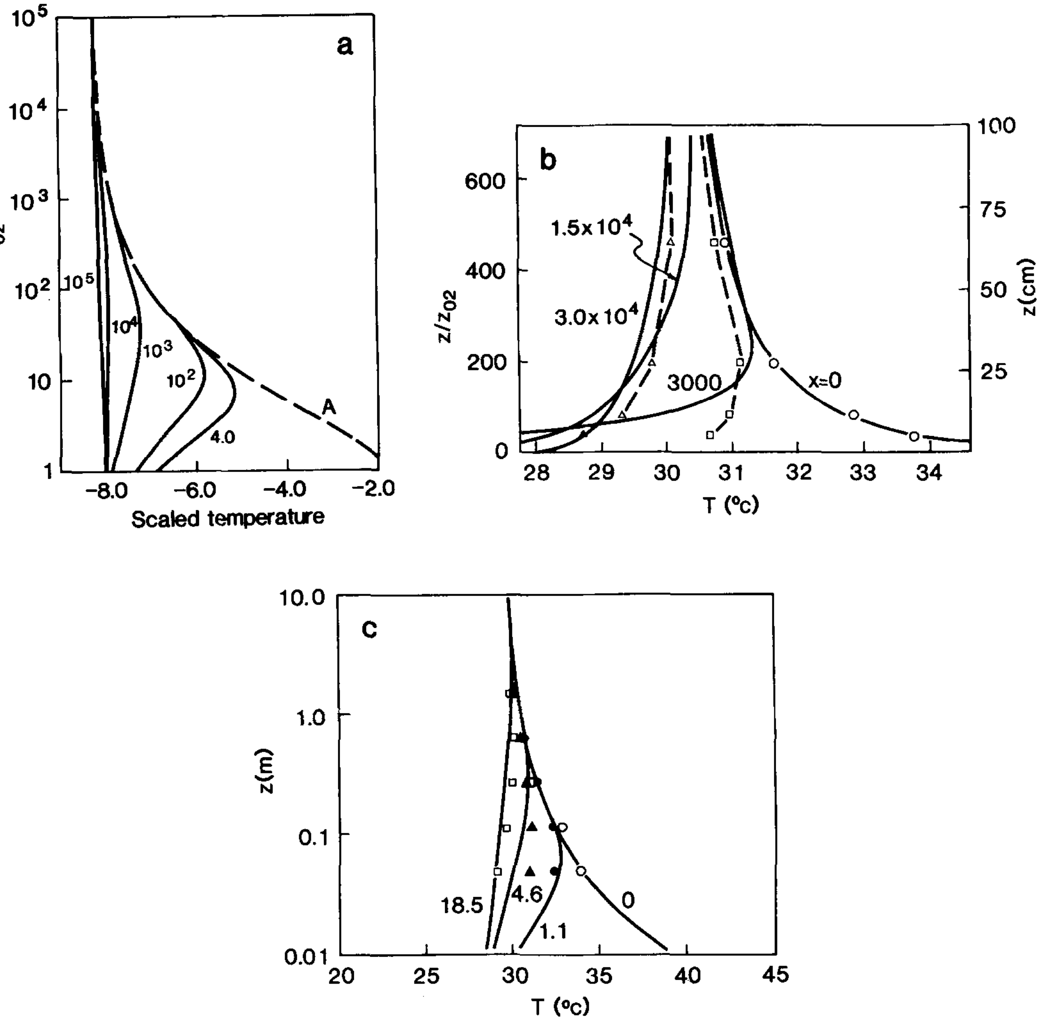

Fig. 8. (a) Small-scale modification of temperature profiles downstream of a surface temperature change, based on the numerical results of Taylor (1971); in this case, $z_{02} = 0.1$ m. Values of x/z_{02} are shown against each curve; curve A has $x = 0$. (b) Comparison of the experimental data of Rider *et al.* (1963) (dashed curves) and the temperature profiles predicted by Taylor (1971) (continuous curves), for $x = 0$ (○), 4.88 (□) and 19.5 m (△), downstream of a roughness and surface evaporation change. Values of x/z_{02} are shown against each theoretical curve. (c) Comparison of the experimental data of Rider *et al.* (1963) and the temperature profiles predicted by Rao *et al.* (1974b), for $x = 0$ (○), 1.15 (●), 463 (▲) and 18.5 m (□). From Rao *et al.* (1974b).

unstable flow downwind of both temperature and heat flux changes were considered, for neutral upstream flow; the behaviour of vertical profiles of τ, u and θ, and of surface stress and heat flux with increasing x/z_0 was described. Simulations for flow over a lake were also described. In Taylor (1971), downstream stable flow was analysed for both neutral and unstable upstream flow, for temperature and heat flux steps at the surface. For illustration, Figure 8a shows simulated temperature profiles for a range of situations relevant to the later discussion on the coastal thermal IBL; and in Figure 8b, both observed (from Rider *et al.* 1963) and predicted temperature profiles illustrate moderate agreement only (compare with Figure 7a).

In the higher order closure modelling of Rao *et al.* (1974b), advection from a dry smooth area to a wet, rougher area was studied and results compared with observations. In addition, they studied the dependence on the downstream θ and q profiles of surface humidity, upstream stability and the magnitude of the roughness change. Comparisons between the numerical predictions of Rao *et al.* and the data of Rider *et al.* (1963) are shown in Figure 8c. Reasonable agreement is achieved, and the importance of thermal stability changes on leading-edge diffusion problems emphasised.

6. Mesoscale Flow and IBL Growth

A number of studies (e.g., Taylor, 1969b; Jensen, 1978; Larsen *et al.*, 1982) related to small-scale flow over roughness changes have attempted to extend the surface-layer approach to the mesoscale. In this, the main problem has been concerned with IBL growth to the full depth of the downstream planetary boundary layer. Such approaches have been reasonably successful in estimating the fetch-height ratio at large fetches based on the relations described earlier for small-scale flow (Section 4), mainly because of the apparent lack of rotation of the boundary-layer wind until the equilibrium IBL depth is achieved. However, so far as detailed IBL structure is concerned, no study has yet critically examined the validity of such an extension.

Most interest at the mesoscale has focussed on the structure and growth of the thermal IBL, and has stemmed from the perceived relevance of the IBL to diffusion and pollution problems in the coastal region. Although the advection of air across the coastline relates to both surface roughness and temperature changes, the primary consideration is often the response and growth of an IBL to a marked step-change in surface temperature. It is recognised that in real-world situations, roughness changes also are present. Theoretical and numerical work discussed above can be applied to this new problem, though IBL behaviour at far greater x (and x/z_0) than considered earlier is often under study. In particular, the emphasis has been on the growth equation for the IBL depth, and the factors affecting the height, so that somewhat different approaches to those described earlier have been used to derive suitable expressions for h. Most of the work until recently concerned the convective thermal IBL, although the stable case has received increased attention in recent years – we delay its discussion until the next main section.

6a. The convective thermal IBL – observations

Apart from the comprehensive observational and numerical study of Durand *et al.* (1989) based on atmospheric observations and a third-order turbulence closure model, and the observational study of Gamo *et al.* (1982) related to the sea breeze, most wind-tunnel and field data analyses to be found in the literature are concerned primarily with IBL growth.

In the wind-tunnel study of Meroney *et al.* (1975), mixed-layer growth is supported by a square-root dependence of height on the fetch x. The few observations discussed by Raynor *et al.* (1975) for both stable and unstable cases, and the detailed data set described by Raynor *et al.* (1979) for onshore flow from cool sea to warm land, confirm this power dependence. Additional support is found in data described by Van der Hoven (1967), Weisman and Hirt (1975) and Kerman *et al.* (1982), and in Hsu (1986) who analysed observations of Druilhet *et al.* (1982), Smedman and Högström (1983) and Ogawa and Ohara (1985). In all of the above, fetches tend to range between several km (e.g., in Hsu, 1986) to about 50 km (e.g., in Raynor *et al.*, 1979 and Durand *et al.*, 1989). In Raynor *et al.* (1975, 1979) observed heights were found to agree with an empirical model derived from physical and dimensional reasoning, viz.,

$$h^2 = C_D \gamma^{-1}(\theta_1 - \theta_s)x \,, \tag{26}$$

where C_D is a low-level drag coefficient over land (in the downstream direction), γ is a lapse rate above the IBL and $\theta_1 - \theta_s$ is the temperature difference between the land surface and the sea. The above equation was simplified by Hsu (1986) who found several sets of data satisfied $h = 1.9x^{1/2}$, with h and x in metres.

In contrast to the above, Gamo *et al.* (1982) studied TIBL structure (for $x < 15$ km) during sea-breeze events on the eastern coast of Japan and found that the TIBL top defined from the θ profiles was well below the level of minimum turbulent kinetic energy. This suggested that entrainment should be significant for the growth of the convective IBL. Durand *et al.* (1989) described detailed aircraft observations of the mean and turbulence structure of the TIBL for offshore flow during the COAST experiment. Fetches were limited to $x < 45$ km. The IBL was revealed as a fairly well-mixed layer, with large horizontal gradients in θ and turbulent kinetic energy for several tens of km inland from the coast. Although these authors identified the top of the IBL in several of their figures, implying growth as $x^{1/2}$, the criteria upon which evaluation of the top was made are not clear. As in the study of Gamo *et al.*, turbulence was still significant above the strongly stable region defining the TIBL top.

At very large values of x, boundary-layer heights must tend towards some equilibrium value, a point discussed by Venkatram (1986) in the context of Equation (26). That is, a square-root dependence must ultimately be invalid at large enough x if h tends to a constant value. This will be discussed later in the section.

6b. The convective thermal IBL – models of IBL development

A slab model approach based on mixed-layer dynamics (e.g., Carson and Smith, 1974) has been used (Venkatram, 1977) to derive a set of governing equations from which $h(x)$ can be determined. Equations for mixed-layer wind components and temperature, the continuity equation and an equation for h allow for numerical solution, with appropriate boundary conditions. The h equation required for closure results from applying the TKE equation at the inversion level and utilising a suitable entrainment assumption (Tennekes, 1973). Venkatram used this numerical model to study the impact of several parameters on growth, including Richardson number (see Figure 9a) and surface roughness (see Figure 9b). For practical application, Venkatram then went on to derive a simplified model, based on vertical-averaging steady-state of the θ equation (Equation (23)), with vertical advection assumed negligible. This gives,

$$\rho c_p h\hat{u}\ \partial\theta/\partial x = H_0 - H_h\,, \tag{27}$$

where the circumflex denotes a vertical average from the surface to h, and H is

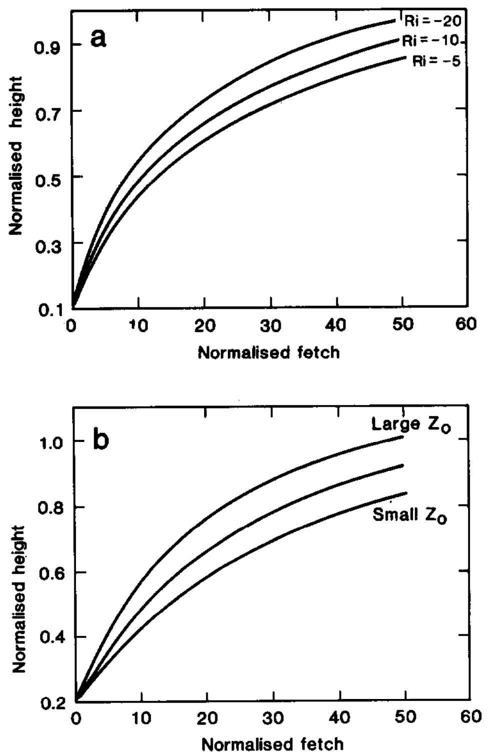

Fig. 9. Dependence of convective IBL growth (in terms of a normalised IBL height) on Richardson number (a) and surface roughness (b); normalised quantities are shown, using predictions from Venkatram (1977) – all length scales are normalised by the quantity $\Delta\theta/\gamma$. From Venkatram (1977).

the heat flux at the surface (H_0) and at $h(H_h)$. With $\Delta\theta$ equal to the temperature jump across the inversion (at h), and γ equal to the lapse rate above h (that is, in the upstream flow), Venkatram defined an entrainment relation

$$\Delta\theta = F\gamma h \, , \tag{28}$$

with F a fraction, such that the ratio of heat fluxes $\beta = -H_h/H_0$ could be written in terms of F, as

$$\beta = F/(1-2F) \, . \tag{29}$$

With the relation between h and $\hat{\theta}$ deduced from

$$\gamma = (\hat{\theta} + \Delta\theta - \theta_s)/h \, , \tag{30}$$

h is given by

$$h = \gamma^{-1}(\hat{\theta} - \theta_s)/(1-F) \, . \tag{31}$$

Combining Equations (27), (28) and (31), Equation (27) becomes in non-dimensional form,

$$\theta_p \, \partial\theta_p/\partial X = B(1-\theta_p) \, , \tag{32}$$

where $\theta_p = (\hat{\theta} - \theta_s)/(\theta_1 - \theta_s)$, $h_p = h\gamma/(\theta_1 - \theta_s)$, $X = x\gamma/(\theta_1 - \theta_s)$ and B = a constant (determined by the value of F and a heat transfer coefficient).

For small X, the solution

$$\theta_p = 1 - \exp(-(\theta_p + BX)) \, , \tag{33a}$$

and

$$h_p = \theta_p/(1-F) \, , \tag{33b}$$

gives h as

$$h^2 = 2\gamma^{-1}(1-2F)^{-1}C_D(\theta_1 - \theta_s)x \, . \tag{34}$$

This is of the same form as Equation (26), illustrating the square root dependence; it needs to be emphasised that this relation is appropriate to the convective thermal IBL and not necessarily applicable to the stable case (see later).

Stunder and Sethuraman (1985) reviewed a number of like formulations, not all with as good a physical basis as the one summarised above. Of seven formulations considered (Van der Hoven, 1967; Plate, 1971; Peters, 1975; Raynor *et al.*, 1975; Weisman, 1976; Venkatram, 1977 and Lyons, 1977), all but one (that of Peters) showed an $x^{1/2}$ behaviour. In addition, all but one (that of Van der Hoven) had a direct dependence on a temperature difference or surface heat flux, i.e., consistent with expectations for convective conditions. Statistical comparisons of the seven schemes, using two data sets of observed thermal IBL heights and other relevant information, were made; these gave the formulation of Weisman (1976) as best predicting the depth. This method closely follows that of Venkatram; starting with the θ equation for steady state, and integrating between the surface and h, assuming no entrainment heat flux, gave

$$h^2 = (2H_0/\rho c_p \gamma u)x \,. \tag{35}$$

Equations (34) and (35) are very similar, particularly if H_0 is replaced by the usual bulk transfer relation. Hanna (1987) pointed out the undesirable asymptotic behaviour of both these equations, viz., that h attains unrealistically high values as $\gamma \to 0$, and $h \to 0$ as $H_0 \to 0$. His suggested empirical linear form, however, should be seen as a purely engineering relation describing several data sets for $x < 15$ km only.

Venkatram (1986) discussed other disadvantages of Equations (34) and (35); in particular, the assumption that H_0 over land is constant in the downstream direction, and hence the implication that h will not achieve the observed equilibrium height (h_e) at large x. He suggested a more realistic model, based on the empirical relation

$$h^2 = h_e^2(1 - \exp(-x/bh_e)) \,, \tag{36}$$

so that $h = h_e$ for large x, and $h^2 = h_e x/b$ for small x. The height h_e is given by usual mixed-layer models, so that Equation (34) is readily recovered.

In summary, Equations (34) to (36) give the convective TIBL depth for onshore flow consistent with the observed behaviour; h increases with greater land roughness and greater temperature difference, and decreases with greater stability above the IBL, for a given fetch x.

6c. The convective thermal IBL – numerical simulations

Durand *et al.* (1989) used a 2D version of a third-order turbulence closure model to study the TIBL structure and growth during daytime, onshore flow. Comparisons were made with their detailed aircraft observations summarised in Section 6a above. In contrast with these observations, and also those of Gamo *et al.* (1982), the TIBL top defined in terms of the θ inversion layer was found to coincide with the level of near-zero heat flux and turbulent kinetic energy, and with maximum temperature variance. In the range of x studied ($x < 50$ km), both observations and numerical results showed h behaving as $h \cong 5x^{1/2}$ (h and x in metres), consistent with Equations (34) and (35) when the appropriate values of H_0, γ, F and wind speed were used (compare this with the results of Hsu (1986) at smaller x – see Section 6a above). This model behaviour was found with clouds absent in the simulations; with their inclusion, growth was more rapid, apparently due to the influence of latent heat release.

6d. The stable thermal IBL – observations

As discussed in the previous section, much of the interest in the stable case is related to offshore flow in the coastal region, from warm land to cool sea. Growth of the stable thermal IBL has recently been studied by appeal to some historical data and dimensional analysis (Mulhearn, 1981; Hsu, 1983), by use of numerical and simple physical models (Garratt, 1987), and by analysis of detailed low-level aircraft data (Garratt and Ryan, 1989). Growth rates are found to be small, with fetches of several hundreds of kilometres required to develop an IBL several hundreds of metres deep. The depth h is found to depend upon $x^{1/2}$ as in the convective case, and is therefore not supportive of small-scale studies which suggest a varying power dependence with thermal stability.

Mulhearn (1981) analysed measurements of θ and q profiles, and wind speed at 300 m height, made in offshore flow over Massachusetts Bay in the USA (see references contained therein). For fetches in the range 5 to 100 km, he used dimensional analysis to collapse the data into the form

$$h \cong 0.015u(g\Delta\theta/\theta)^{-1/2}x^{1/2}, \tag{37}$$

where u is a wind speed near the IBL top, $\Delta\theta$ is the temperature difference between the sea surface and upstream air and g is the acceleration due to gravity. In addition, he was able to describe the form of the temperature profiles (for $x \cong 15$ km and $x/z_0 \cong 200$) as $(\theta - \theta_s)/\Delta\theta = (z/h)^{1/4}$, a profile shape quite different from that observed by Garratt and Ryan (1989) – see later.

Hsu (1983) analysed several sets of data (for the sources of these data sets, see references contained therein) using as a theoretical framework a growth relation found by combining the results of Venkatram (1977) for the convective IBL and that of Mulhearn (1981). Garratt (1987) has questioned the use of Venkatram's formulation in the stable case (Equation (34)); this equation shows $h \propto \Delta\theta^{1/2}$, whereas the stable IBL should behave as $h \propto \Delta\theta^{-1/2}$ consistent with energetic considerations. Hsu's data analysis, covering a fetch range from 5 to 500 km, gave

$$h \cong 0.57x^{1/2}, \tag{38}$$

where the numerical factor of 0.57 compares with 1.9 in the convective case described in Hsu (1986) (but for an x range from 20 m to 8 km only), and with about 5 found in Durand *et al.* (1989) for $x < 50$ km.

Some of the most detailed mean flow and turbulent observations of the stable IBL were described recently by Garratt and Ryan (1989), for offshore flow situations in southeast Australia. The dependence of the IBL depth on a range of external parameters, including the temperature difference $\Delta\theta$, the geostrophic wind and the fetch, was discussed in the context of the study of Garratt (1987) – see next section. The nature of the θ profiles over the sea within the IBL was found to be quite different to that found in the stable boundary layer over land. Over the sea, the θ profiles were found to have large positive curvature with vertical gradients increasing with height, interpreted as reflecting the dominance of turbulent cooling within the layer. The behaviour is consistent with known behaviour in the nocturnal boundary layer over the land, where curvature becomes negative (vertical gradients decreasing with height) as radiative cooling becomes

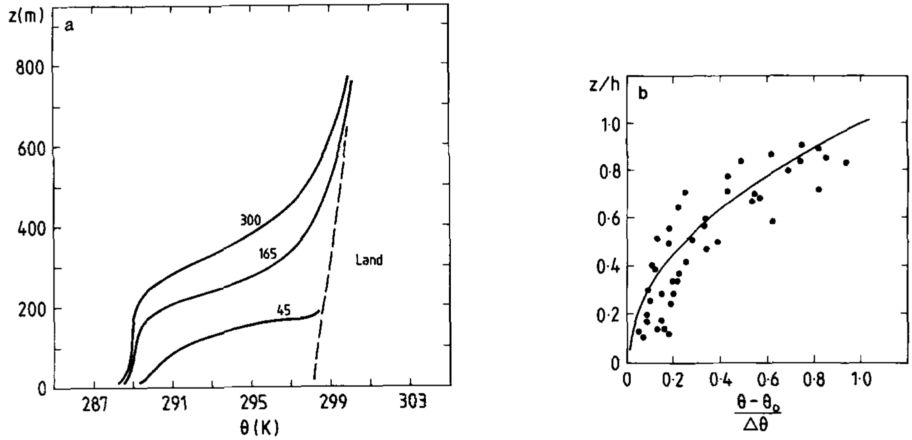

Fig. 10. Temperature profiles observed within 300 km of the coast in a stable IBL with offshore flow – data from Garratt and Ryan (1989). Absolute (a) and normalised (b) temperature profiles are shown; values of x in km are indicated in (a).

dominant. Thus, Garratt and Ryan (1989) found

$$(\theta - \theta_s)/\Delta\theta = (z/h)^2\,, \tag{39}$$

in contrast with the result of Mulhearn (1981) at smaller fetches, with no significant x/h dependence for the x/h range between 300 and 1800. This suggested an approximate self-preserving form of the temperature profile at large x, examples of which are given in Figure 10 in absolute and normalised form (values of x are indicated). At, and just above h, there is a region of marked negative curvature so that $\partial\theta/\partial z$ tends to small values aloft. The comparison with the normalised profiles of Mulhearn (1981 – see his Figure 4) suggests that at smaller values of x/h (or

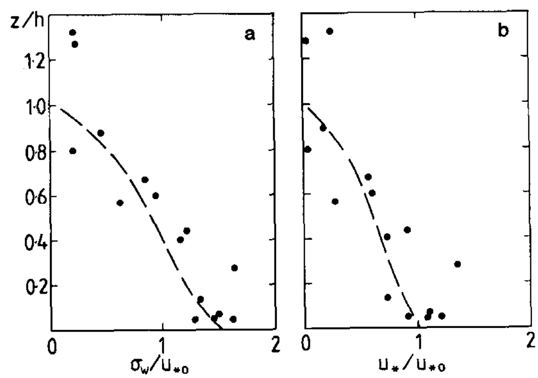

Fig. 11. Normalised turbulence velocity quantities – vertical velocity (a) and friction velocity (b) – as a function of z/h for x between 50 and 300 km; u_{*0} is the surface friction velocity. Curves are from Caughey *et al.* (1979) for the stable boundary layer over land; data are from Garratt and Ryan (1989).

x/z_0), profile curvature changes rapidly, and the assumption of a self-preserving profile form is not valid throughout the TIBL at small fetches.

Garratt and Ryan (1989) also discussed the turbulent structure within the IBL, in terms of non-dimensional quantitites normalised by the suface friction velocity, as functions of z/h. Vertical profiles of several such quantities – velocity variances and stress, together with the normalised wavelength of the spectral maximum, agreed well with known structure for the stable boundary layer over land (Caughey *et al.*, 1979). This is illustrated in Figure 11 for the case of vertical velocity variance and friction velocity.

6e. The stable thermal IBL – numerical

The main study has been that of Garratt (1987) who used a mesoscale numerical model to investigate the internal structure and growth of the IBL beneath warm, continental air flowing over a cooler sea. Both steady-state and diurnally-varying offshore flow were considered. Overall, the mean profiles of wind and temperature behaved similarly to those found in the stable layer over land; in particular, well away from the coast, the IBL was found to have a near-critical value of the layer-flux Richardson number of 0.18. This fact is important in the development of a physically-based model of IBL growth.

6f. The stable thermal IBL – model of IBL growth

Relations (37) and (38) are mainly based on dimensional analysis, but Equation (37) was given a firm physical basis in the analysis of Garratt (1987) and Garratt and Ryan (1989; see their appendix). The starting point is Equation (23), integrated between the surface and h, with an assumed linear flux profile and assumed self-preservation forms for the profiles, viz., $u/U = f_1(z/h)$, $(\theta - \theta_s)/\Delta\theta = f_2(z/h)$ and $w/w_h = f_3(z/h)$. Here w_h is the vertical velocity at h and U is a large-scale wind (or geostrophic) component normal to the coast. Note that, in contrast to the analysis of Venkatram (1977) for the convective case, vertical advection is included in this analysis (compare with Equation (23)). Assuming that $\partial\theta/\partial x \cong -(\partial\theta/\partial z)(\delta h/\delta x)$, and that the entrainment heat flux (H_h) is negligible, it can be readily shown that,

$$\partial h/\partial x = w_h/U + (A_0/\rho c_p U \Delta\theta) H_0 \,. \tag{40}$$

In the above, A_0 is a profile shape factor depending on f_1 and f_2. The heat flux is now found through use of a critical layer-flux Richardson number (R_f) concept. Again assuming self-similar profile forms, $H = H_0 f_4(z/h)$ and $u_*^2 = u_{*0}^2 f_5(z/h)$, H_0 is given by

$$H_0 = \rho c_p u_{*0}^2 G R_f / ((g/\theta) h f(z/h)). \tag{41}$$

Here G is a geostrophic wind, at an angle β to the offshore wind U (hence $U = G \cos\beta$), and $f(z/h)$ is a function of f_1, f_4 and f_5. Combining Equations (40) and

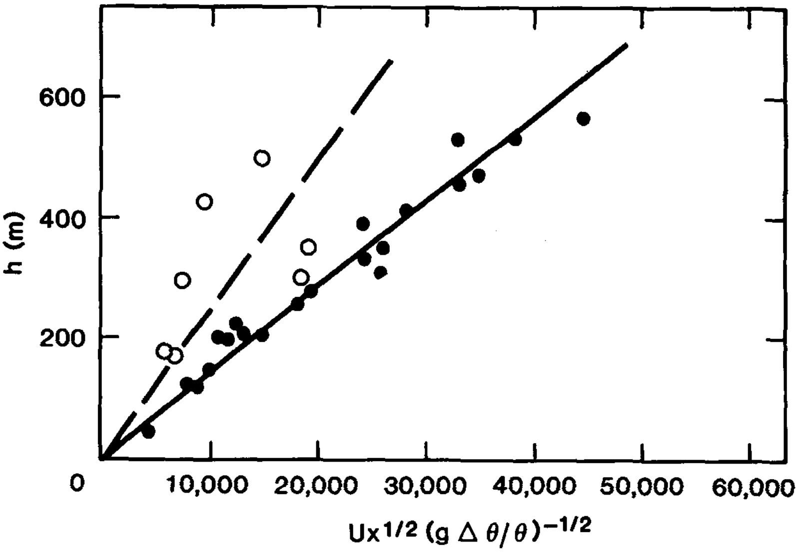

Fig. 12. Numerical results of IBL depth (closed circles) based on the offshore flow simulations discussed in Garratt (1987), compared with predictions of Equation (42) – for values of x between 100 and 900 km. Open circles represent the observations described in Garratt and Ryan (1989). The solid and dashed lines represent the best 'least squares' linear fit to the model and observational results, respectively.

(41), and integrating gives

$$h^2 = \alpha_1 U^2 (g\Delta\theta/\theta)^{-1} x \,, \tag{42}$$

where

$$\alpha_1 = 2A_0 f(z/h) R_f C_D / \cos^3 \beta \tag{43}$$

with C_D defined by $u_{*0}^2 = C_D G^2$. Figure 12 shows numerical results and observations which gave $\alpha_1^{1/2} \cong 0.014$ (numerical) and $\cong 0.024$, respectively – these values can be compared with that implied in Equation (37) based on the results of Mulhearn (1981); see also the discussion in Hsu (1989). The analysis leading to the result expressed by Equations (42) and (43) suggests that the value of α_1 will depend on the angle between the geostrophic wind and the coastline normal, even if U is replaced by G in Equation (42). To avoid this problem, x would have to be replaced by the actual distance from the coast along the geostrophic wind axis; it is probable that differences between various sets of numerical calculations and observations are partly related to the existence of a non-zero β.

7. Summary

Studies of the IBL are often related to associated problems of local and mesoscale advection; leading edge effects; boundary-layer growth; and the fetch-height ratio of interest in micrometeorological studies. This review has attempted to summarize in a logical sequence much of the western literature dealing with the atmospheric IBL published in the last few decades (with some reference to relevant laboratory studies).

Major interest has centred on the problem of small-scale neutral flow over a step change in surface roughness; the observations of Bradley (1968) are still the most comprehensive set available and are widely used for comparison with theoretical and numerical results. In the related problems of non-neutral flow over step changes in roughness, surface temperature, or heat and moisture flux, much emphasis has been placed on comparisons with the micrometeorological observations of Rider *et al.* (1963). In both cases, downstream fetches are limited to a few tens of metres. In the small-scale context, with fetches less than a few km, height-fetch ratios tend to range between about 1/10 for the IBL top, to about 1/200 for the top of an inner equilibrium layer at which 90% adjustment to local surface conditions has occurred.

The larger-scale problem of mesoscale advection has been mainly concerned with flow across a coastline, mainly because of its relevance to problems of pollution in coastal regions. In the convective case, usually with onshore flow from a cool sea to warmer land, full boundary-layer development occurs with fetches of less than 50 km. In the stable case, with offshore flow from warm land to cool sea, the IBL may be only a few hundreds of metres deep after fetches of 500 km or more. Physically based formulations for the IBL depth suggest height-fetch ratios of about 1/10 and 1/2000 for the convective and stable cases. respectively.

At the small scales in the atmosphere, and in the wind tunnel, the neutral IBL is found to grow as $x^{4/5}$ approximately, for a smooth-to-rough transition, with a slightly slower growth for rough-to-smooth flow. The effects of unstable stratification give a more rapid growth, with a dependence close to $x^{1.4}$ in very unstable conditions according to numerical results. At the larger scale, growth follows an $x^{1/2}$ dependence for both unstable and stable stratification, with direct, or inverse, dependence on air-sea temperature difference, large-scale wind and surface roughness.

Much is now known of the nature of the atmospheric IBL at the small scale; theories, integral methods and numerical approaches all have a contribution to make, but there has been too much reliance on a rather small data bank for validation. The latter mainly comprises the observations of Rider *et al.* (1963), Dyer and Crawford (1965) and Bradley (1968); there would seem to be a requirement for an additional comprehensive observational set, perhaps covering a somewhat larger fetch range to encompass the transition between the small-scale problem and that at the mesoscale.

In the coastal context, there would appear to be some uncertainty regarding the shape, and evolution with fetch, of the temperature profile within the stable IBL in offshore flow, Such a problem would seem to be worthy of further study, either through observations or the numerical modelling approach. The coastal situation in particular also presents a problem that has been relatively unexplored in the literature – that of the thermal wind and its interaction with the IBL. The use of mesoscale models with realistic boundary-layer parameterization schemes could contribute significantly here, particularly regarding the role of sea-breeze-type circulations.

Finally, a major unsolved problem concerns the boundary-layer response to multiple step changes in surface properties – of roughness, temperature or moisture. A good illustration of this is the nature of patchwork landscapes, and how such surface heterogeneities should be represented in surface parameterization schemes in numerical models of the atmosphere.

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